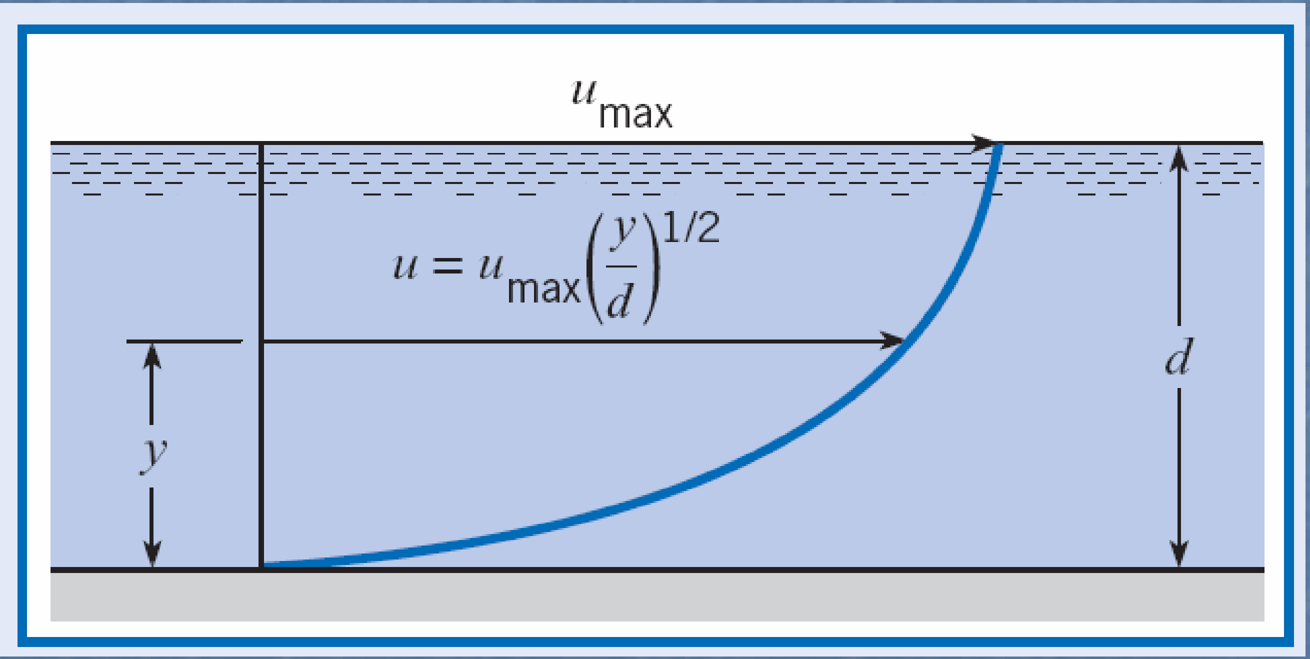
Example (5.3)

The water velocity in the channel shown in the accompanying figure has a distribution across the vertical section equal to $u/u_{max} = (y/d)^{1/2}$. What is the discharge in the channel if the channel is 2 m deep ($d = 2$ m) and 5 m wide and the maximum velocity is 3 m/s?

$$\dot{Q} = ?$$



Example (5.3)

Solution The discharge is given by

$$Q = \int_0^d u\,dA$$

The channel is 5 m wide, so the differential area is 5 *dy*. Thus

$$Q = \int_0^2 u_{\max}(y/d)^{1/2} 5\,dy$$

$$= \frac{5u_{\max}}{d^{1/2}} \int_0^2 y^{1/2}\,dy$$

$$= \frac{5u_{\max}}{d^{1/2}} \frac{2}{3} y^{3/2} \Big|_0^2$$

$$= \frac{5 \times 3}{2^{1/2}} \times \frac{2}{3} \times 2^{3/2} = 20\ \mathrm{m^3/s}$$

◁

CONTROL VOLUME APPROACH

PROBLEM 5.2

Situation: Water flows in a 16 in pipe. $V = 3$ ft/s.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (3 \text{ ft/s})(\pi/4 \times 1.333^2) \\ & \boxed{Q = 4.19 \text{ ft}^3/\text{s}} \\ &= (4.17 \text{ ft}^3/\text{s})(449 \text{ gpm/ft}^3/\text{s}) \\ & \boxed{Q = 1880 \text{ gpm}} \end{aligned}$$

CONTROL VOLUME APPROACH

PROBLEM 5.4

Situation: An 8 cm. pipe carries air, $V = 20$ m/s, $T = 20^{o}$C, $p = 200$ kPa-abs.

Find: Mass flow rate: $\dot{m}$

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 200,000/(287 \times 293) \\ \rho &= 2.378\ kg/m^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho V A \\ &= 2.378 \times 20 \times (\pi \times 0.08^2/4) \end{aligned}$$

$$\boxed{\dot{m} = 0.239 \text{ kg/s}}$$

PROBLEM 5.8

Situation: In a circular duct the velocity profile is $v(r) = V_o(1 - r/R)$, where V_o is velocity at $r = 0$.

Find: Ratio of mean velocity to center line velocity: $\bar{V}/V_o$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$Q = \int v dA$$

where $dA = 2\pi r dr$. Then

$$\begin{aligned} Q &= \int_0^R V_0(1-(r/R))2\pi r dr \\ &= V_0(2\pi)((r^2/2) - (r^3/(3R))) \mid_0^R \\ &= 2\pi V_0((R^2/2) - (R^2/3)) \\ &= (2/6)\pi V_0 R^2 \end{aligned}$$

Average Velocity

$$\begin{aligned} \bar{V} &= \frac{Q}{A} \\ \frac{\bar{V}}{V_0} &= \frac{Q}{A}\frac{1}{V_0} \\ &= \frac{(2/6)\pi V_0 R^2}{\pi R^2}\frac{1}{V_0} \end{aligned}$$

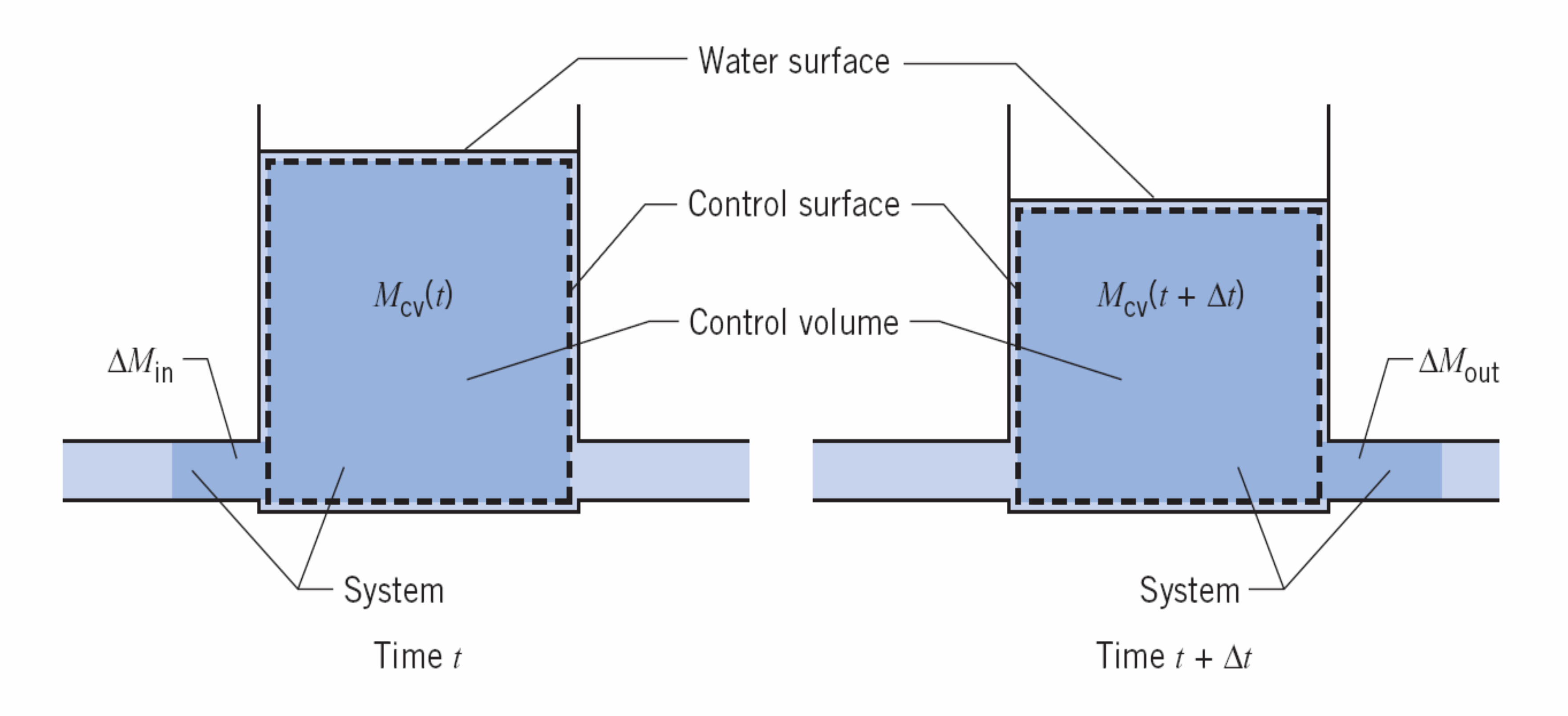
$$\boxed{\bar{V}/V_o = 1/3}$$

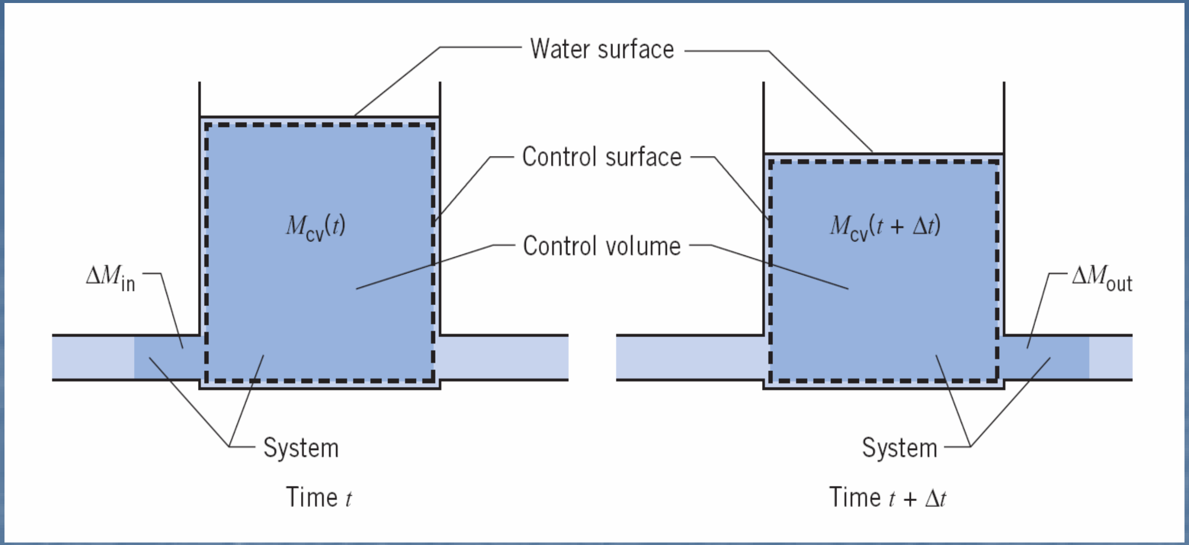
CONTROL VOLUME APPROACH

a. **System:** The mass of a system is constant

b. **Control Volume** (*CV*) : Is defined as a volume in space

c. **Control Surface** (*CS*) : is the surface enclosing the control volume.





$$M_{sys}(t) = M_{cv}(t) + \Delta M_{in}$$

$$M_{sys}(t+\Delta t) = M_{cv}(t+\Delta t) + \Delta M_{out}$$

By definition, the mass of the system is constant

i.e. $M_{cv}(t+\Delta t) + \Delta M_{out} = M_{cv}(t) + \Delta M_{in}$

$$\Delta M_{cv} = \Delta M_{in} - \Delta M_{out}$$

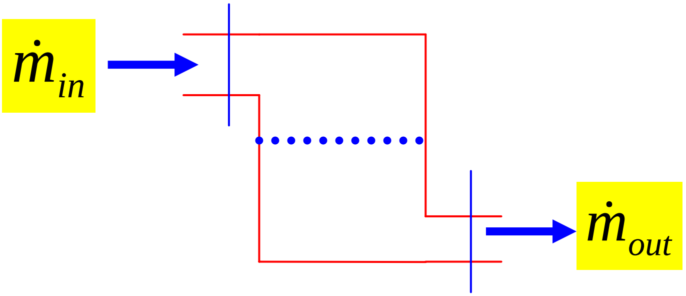
In the limit, $\dfrac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

Example 5.4 (P149)

Area of a tank $= 10\, m^2$

$\dot{m}_{in} = 7\, kg/s$

$\dot{m}_{out} = 5\, kg/s$



Find the rate at which the water level in the tank changing? $\left(\frac{dh}{dt}\right)$

$$\frac{dM_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{d}{dt}(\rho A h) = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{dh}{dt} = \frac{\dot{m}_{in} - \dot{m}_{out}}{\rho A} = \frac{7-5}{1000 \times 10} = 0.0002\, m/s = 0.72\, m/h$$

CONTROL VOLUME APPROACH

END OF LECTURE (2)